Properties of the Real Numbers

The following are the properties of addition and multiplication if \( x \), \( y \), and \( z \) are real numbers:

<table>
<thead>
<tr>
<th>Properties</th>
<th>Addition</th>
<th>Multiplication</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commutative</td>
<td>( x + y = y + x )</td>
<td>( x \cdot y = y \cdot x )</td>
</tr>
<tr>
<td>Associative</td>
<td>((x + y) + z = x + (y + z))</td>
<td>((x \cdot y) \cdot z = x \cdot (y \cdot z))</td>
</tr>
<tr>
<td>Identity</td>
<td>( x + 0 = x )</td>
<td>( x \cdot 1 = x )</td>
</tr>
<tr>
<td>Inverse</td>
<td>There is a unique number (-x) such that ( x + (-x) = 0 )</td>
<td>If ( x \neq 0 ), there is a unique number ( \frac{1}{x} ) such that ( x \cdot \frac{1}{x} = 1 )</td>
</tr>
</tbody>
</table>

| Distributive     | \( x \cdot (y + z) = x \cdot y + x \cdot z \)                           | Multiplication by zero                         |
|                 |                                                                           | \( x \cdot 0 = 0 \)                            |

**Commutative Property:** When adding or multiplying two numbers, the order of the numbers can be reversed without changing the result.

- **Addition:** \( 3 + 5 = 5 + 3 \) now check! \( 3 + 5 = \_\) and \( 5 + 3 = \_\)
- **Multiplication:** \( 4 \cdot 7 = 7 \cdot 4 \) now check! \( 4 \cdot 7 = \_\) and \( 7 \cdot 4 = \_\)

**Associative:** When adding or multiplying three or more numbers, the result does not change if the numbers are grouped differently.

- **Addition:** \((1 + 2) + 3 = 1 + (2 + 3)\) now check! \((1 + 2) + 3 = (\_\) + 3 = \_\) and \( 1 + (2 + 3) = 1 + (\_) = \_\)
- **Multiplication:** \((1 \cdot 2) \cdot 3 = 1 \cdot (2 \cdot 3)\) now check! \((1 \cdot 2) \cdot 3 = (\_\) \cdot 3 = \_\) and \( 1 \cdot (2 \cdot 3) = 1 \cdot (\_) = \_\)

**Identity:** Addition and multiplication each have an identity element. This is a special number that does not change the value of other numbers when combined. For addition this number is zero, and for multiplication the number is one.

- **Addition:** \( 5 + 0 = \_\)
- **Multiplication:** \( 5 \cdot 1 = \_\)

**Inverse:** Addition and multiplication each have a unique inverse element for each real number (except zero for multiplication!) A number combined with its inverse gives the identity element.

- **Addition:** \( 5 + (-5) = \_\)
- **Multiplication:** \( 5 \cdot \frac{1}{5} = \_\)

**Distributive:** We say that multiplication distributes over addition of real numbers.

- \( 2 \cdot (1 + 3) = 2 \cdot 1 + 2 \cdot 3 \) now check! \( 2 \cdot (1 + 3) = 2 \cdot (\_) = \_\) and \( 2 \cdot 1 + 2 \cdot 3 = \_ + \_ = \_\)

**Addition does not distribute over multiplication!**

- \( 2 + (1 \cdot 3) \neq (2 + 1) \cdot (2 + 3) \) because \( 2 + (1 \cdot 3) = 6 \) and \( (2 + 1) \cdot (2 + 3) = 15 \)

**Multiplication by zero:** Any real number multiplied by zero is equal to zero.

\( 5 \cdot 0 = \_\)