Multiplying and Dividing Rational Expressions

Purpose:
This is intended to review and practice the skill of multiplying and dividing rational expressions.

Activity: Work through the following activity/problems. You may use your book to help you, but do not consult a tutor until you have made valid attempts on all of the problems in this activity.

When multiplying rational expressions, remember to factor first. Then, multiply across the top and multiply across the bottom. Finally, divide out the common factors.

Example:
\[
\frac{x^2 - 9}{2x^2 - x - 10} \cdot \frac{4x^2 - 25}{x^2 + 4x - 21} = \frac{(x - 3)(x + 3)}{(2x - 5)(x + 2)} \cdot \frac{(2x - 5)(2x + 5)}{(x - 3)(x + 7)}
\]

Factor.

\[
= \frac{(x - 3)(2x - 5)(x + 3)(2x + 5)}{(x - 3)(2x - 5)(x + 2)(x + 7)}
\]

Multiply numerators and denominators.

\[
= \frac{(x + 3)(2x + 5)}{(x + 2)(x + 7)}
\]

Divide out common factors.

When dividing rational expressions, we first factor. Then we must multiply by the reciprocal of the second number first. Finally, multiply and divide out common factors.

Example 1:
\[
\frac{x^2 - 4}{x^2 + x - 6} \div \frac{x^2 + 5x + 6}{2x} = \frac{(x + 2)(x - 2)}{(x + 3)(x - 2)} \div \frac{(x + 3)(x + 2)}{2x}
\]

Factor.

\[
= \frac{(x + 2)(x - 2)}{(x + 3)(x - 2)} \cdot \frac{2x}{(x + 3)(x + 2)}
\]

Multiply by reciprocal of 2nd number.

\[
= \frac{(x + 2)(x - 2)2x}{(x + 2)(x - 2)(x + 3)(x + 3)}
\]

Multiply numerators and denominators.

\[
= \frac{2x}{(x + 3)(x + 3)} = \frac{2x}{(x + 3)^2}
\]

Divide out common factors.
Exercises. Perform the operations and simplify.

1. \( \frac{-6x^4}{3x^5} \div \frac{(2x^2)^2}{-4} \)

2. \( \frac{3(x-1)}{y} \div \frac{5(x-1)^2}{2y} \)

3. \( \frac{12x + 24}{36x - 36} \div \frac{6x + 12}{8x - 8} \)

4. \( \frac{y^2 - 16}{y + 3} \div \frac{y - 4}{y^2 - 9} \)

5. \( \frac{4y + 12}{2y - 10} \div \frac{y^2 - 9}{y^2 - y - 20} \)

6. \( \frac{2m^2 - 5m - 12}{m^2 - 10m + 24} \div \frac{4m^2 - 9}{m^2 - m + 18} \)

Review: Meet with a tutor to verify your work on this worksheet and discuss some of the areas that were more challenging for you. If necessary, choose more problems from the homework to practice and discuss with the tutor.

For tutor use: Please check the appropriate box.

- Student has completed worksheet but may need further assistance. Recommend a follow-up with instructor.
- Student has mastered topic.
Answers: For Tutor Use Only

1. \( \frac{2}{x^5} \)  
2. \( \frac{6}{5(x-1)} \)

3. \( \frac{4}{9} \)  
4. \( (y+4)(y-3) \)

5. \( \frac{2(y+4)}{y-3} \)  
6. \( \frac{m-3}{2m-3} \)

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