Title: Systems of Linear Equations – Elimination (Addition) Method (Part 1)
Class: Math 100 or Math 107
Author: Jason Miner
Instructions to tutor: Read instructions and follow all steps for each problem exactly as given.
Keywords/Tags: systems, systems of linear equations, elimination, addition, consistent

Systems of Linear Equations – Elimination (Addition) Method

Purpose:
This is intended to refresh your knowledge about solving systems of linear equations using the elimination (addition) method, where there is a single solution.

Recall that a system of equations consists of two or more equations each with two or more variables. A solution to a system in two variables is an ordered pair \((x, y)\) that satisfies each equation in the system. For now, we will concentrate on systems of linear equations.

Elimination (Addition) Method – Add a multiple of one equation to the other in order to eliminate one of the variables. After this is done, you will have a single equation with one variable – solve for it. Then back-substitute to find the other.

Example: Solve \( \begin{cases} x + y = 2 \\ 2x - 3y = 9 \end{cases} \) using the elimination method.

Note that if we multiply the first equation by \(-2\) and add it to the second, the variable \(x\) vanishes:

\[
\begin{align*}
-2(x + y &= 2) \\
2x - 3y &= 9
\end{align*}
\Rightarrow
\begin{align*}
-2x - 2y &= -4 \\
+ 2x - 3y &= 9
\end{align*}
\Rightarrow

-5y = 5
\]

Now that we have eliminated a variable, we may solve for \(y\): \(-5y = 5 \Rightarrow y = -1\).

We can substitute this into one of the original equations to find \(x\).

Let’s use the first equation: \(x + y = 2 \Rightarrow x - 1 = 2 \Rightarrow x = 3\).

So our solution is the ordered pair \((3, -1)\). (Note that this is where the two lines intersect.)
Example: Now it’s your turn. Solve \[
\begin{align*}
3x - 2y &= 6 \\
x + 4y &= 4
\end{align*}
\] using the elimination method.

We have a choice to make here – should we try to eliminate \(x\) or \(y\)?

What would you have to multiply the 2nd equation by to eliminate \(x\)? 

What would you have to multiply the 1st equation by to eliminate \(y\)?

Let’s take the 2nd option: \[
\frac{2(3x - 2y = 6)}{x + 4y = 4}
\]

Did you find that \(x = \frac{16}{7}\)? If not, go back and check your work.

Now go back to one of the original equations and solve for \(y\).

Did you get \((16, 3)\) for your solution? Good! Now try the next two on your own.

Example: Solve using the substitution method.

(a) \[
\begin{align*}
4x - y &= 7 \\
-2x + 3y &= 9
\end{align*}
\]

(b) \[
\begin{align*}
5x + 6y &= 4 \\
2x - 3y &= -3
\end{align*}
\]

(The answers are \((3,5)\) for (a) and \((-\frac{2}{9}, \frac{23}{27})\) for (b). If you did not get these, consult a tutor for help.)