Placement Guide for Math 150: Calculus with Analytic Geometry I

What is Math 150? Math 150 is the first course in the Calculus sequence for STEM majors.

Who should take Math 150? Math 150 is for students whose educational goal requires Calculus for STEM majors. *Some* biological sciences and some economics and business majors should take Math 150 instead of Math 130 depending on their transfer goals; see an academic counselor if you are unsure. You should take Math 150 if your major and transfer goals require it and you assess yourself as *prepared to succeed* in this course after reading through this Guide.

Am I ready for Math 150? What are my options?

On the next page you will find some problems to help you assess your readiness for Math 150. These problems are broken into two categories: College Algebra and Trigonometry. Depending on your comfort level with these problems you have four options:

- **Enroll in Math 150**: Choose this option if you can confidently solve most of the problems from both categories (College Algebra and Trigonometry).

- **Enroll in Math 150 along with Math 150C, a 2-unit Support Course:** Choose this option if most of these topics are familiar to you but *some* of the problems from one or both categories are unfamiliar or would be difficult to complete, or if you could generally use review on a lot of these topics.

- **Enroll in Math 138:** Choose this option if you can confidently solve most of the problems from the College Algebra category but have significant gaps in your Trigonometry skills.

- **Enroll in Math 137 or below:** Choose this option if a lot of the problems from both categories are unfamiliar to you or you feel uncomfortable with most of these skills. In this case, see the Placement Guide for Math 137 to help make your choice:
  http://sbcc.edu/assessmentcenter//files/137placementguide.pdf

Problems to help you assess your readiness for Math 150

Note: You do not necessarily need to try to work through all these problems to completion. It may be enough to read through the problems and their solutions to get a feel for whether you are ready for Math 150.

None of these problems require a calculator and you may be required to demonstrate these skills in Math 150 without the use of a calculator.

College Algebra

1. Factor each expression.

(a) $2x^2 + 5x - 12$ (b) $x^3 - 3x^2 - 4x + 12$

2. Simplify the complex fraction $\dfrac{\frac{y}{x} - \frac{x}{y}}{\frac{1}{y} - \frac{1}{x}}$.

3. Rewrite the expression $\dfrac{h}{\sqrt{4+h} - 2}$ with no radicals in the denominator and then simply as much as possible.

4. Rewriting the following quadratic expression $2x^2 - 12x + 11$ in the form $a(x-h)^2 + k$ by using the technique of completing the square.

5. Find all real solutions of the following equations.

(a) $\dfrac{2x}{x+1} = \dfrac{2x-1}{x}$ (b) $3|x-4| = 10$ (c) $x^{2/3} - x^{1/3} - 6 = 0$

(d) $e^{2x+1} = 10$ (e) $2\ln x = \ln(x+3) + \ln(x-1)$

6. Solve each inequality. Write your answer using interval notation.

(a) $x(x-1)^2(x+2) > 0$ (b) $|x-4| < 3$ (c) $\dfrac{2x-3}{x+1} \le 1$

7. Let $A(-7,4)$ and $B(5,-12)$ be points in the plane.

(a) Find the equation of the line that passes through *A* and *B*.
(b) Find the *x*-intercept and *y*-intercept of the line that you found in (a).
(c) Find the midpoint of the segment *AB*.
(d) Find the length of the segment *AB*.
(e) Find an equation of the circle for which *AB* is a diameter.

8. If $f(x) = x^3$, evaluate the expression $\dfrac{f(2+h) - f(2)}{h}$ and simplify your answer.

9. Let $f(x) = x^2 + 1$, $g(x) = \sqrt{x}$. Find and simplify the following function compositions and state the domain for each.

(a) $(g \circ f)(x)$ (b) $(f \circ g)(x)$

10. The function $y = f(x)$ is graphed below. Use the graph to answer the following:

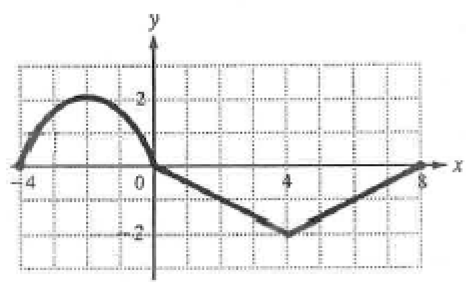
(a) Give the domain and range using interval notation.

(b) On what interval(s) of x-values is f increasing?

(c) On what interval(s) of x-values is $f(x) \le 0$?

(d) Find the value of $f(-2)$ and $f(6)$.

(e) For which value(s) of x is $f(x) = -2$?



11. Let $f(x) = \begin{cases} 1 - x^2 & \text{if } x \le 0 \\ 2x + 1 & \text{if } x > 0 \end{cases}$

(a) Evaluate $f(-2)$ and $f(1)$. (b) Sketch the graph of f.

12. Without using a calculator, make a rough sketch of the graph. You don't need to label the coordinates of specific points – just show the general shape and the end-behavior.

(a) $y = x^3$ (b) $y = (x+1)^3$ (c) $y = (x-2)^3 + 3$

(d) $y = 4 - x^2$ (e) $y = \sqrt{x}$ (f) $y = 2\sqrt{x}$

(g) $y = -2^x$ (h) $y = 1 + x^{-1}$ (i) $y = e^x$

(j) $y = \ln x$

Trigonometry

1. Find the length of *s* as pictured below. Start by converting 140° to radians.

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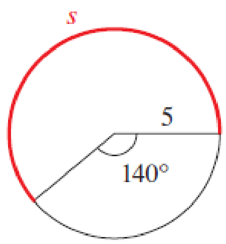
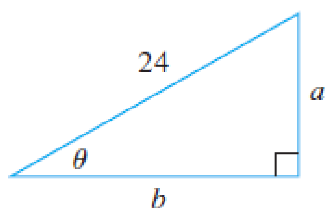


2. Find the values without a calculator.

(a) $\tan\left(\pi/3\right)$ (b) $\sin\left(7\pi/6\right)$ (c) $\sec\left(5\pi/3\right)$

3. Express the lengths of *a* and *b* in the figure in terms of θ.



4. If $\cos x = -2/3$ and $\pi < x < 3\pi/2$, find the exact values of $\cos 2x$ and $\sin 2x$.

For reference: $\cos 2x = \cos^2 x - \sin^2 x$ and $\sin 2x = 2\sin x \cos x$.

5. Express $\cos^4 x$ in terms of trigonometric functions only to the first power.

For reference: $\cos^2 x = \dfrac{1+\cos 2x}{2}$

6. Find all values of *x* such that $\sin 2x = \sin x$ and $0 \le x \le 2\pi$.

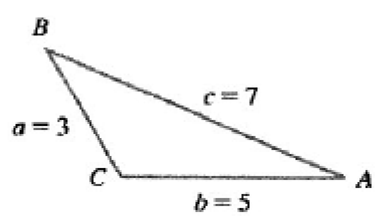
7. Sketch a graph of the function $y = 1 + \sin 2x$ without using a calculator.

8. Evaluate each of the following without a calculator.

(a) $\arccos\left(-\dfrac{\sqrt{2}}{2}\right)$ (b) $\arcsin\left(\sin\dfrac{5\pi}{4}\right)$ (c) $\tan\left(\arccos\dfrac{3}{4}\right)$

9. Find the measure of angle C in the triangle pictured here:

Hint: Use Law of Cosines (which you can look up if you don't remember).



College Algebra Solutions

1. (a) By inspection, trial and error, or the *ac*-method, $2x^2+5x-12=\boxed{(2x-3)(x+4)}$

(b) Using factoring by grouping,

$$x^3-3x^2-4x+12=x^2(x-3)-4(x-3)=(x-3)(x^2-4)=\boxed{(x-3)(x+2)(x-2)}$$

2. $$\frac{\frac{y}{x}-\frac{x}{y}}{\frac{1}{y}-\frac{1}{x}}=\frac{xy\left(\frac{y}{x}-\frac{x}{y}\right)}{xy\left(\frac{1}{y}-\frac{1}{x}\right)}=\frac{y^2-x^2}{x-y}=\frac{(y+x)(y-x)}{x-y}=\boxed{-(y+x)}$$

3.

$$\frac{h}{\sqrt{4+h}-2}=\frac{h}{\sqrt{4+h}-2}\cdot\frac{\sqrt{4+h}+2}{\sqrt{4+h}+2}=\frac{h\left(\sqrt{4+h}+2\right)}{\left(\sqrt{4+h}\right)^2-2^2}=\frac{h\left(\sqrt{4+h}+2\right)}{4+h-4}$$

$$=\frac{h\left(\sqrt{4+h}+2\right)}{h}$$

$$=\boxed{\sqrt{4+h}+2}$$

4. $2x^2-12x+11=2\left(x^2-6x\right)+11=2\left(x^2-6x+9\right)-18+11=\boxed{2(x-3)^2-7}$

5.

(a)

$$\frac{2x}{x+1}=\frac{2x-1}{x}$$

$$x(x+1)\frac{2x}{x+1}=x(x+1)\frac{2x-1}{x}$$

$$x(2x)=(x+1)(2x-1)$$

$$2x^2=2x^2+x-1$$

$$0=x-1$$

$$\boxed{x=1}$$

(b)

$$3|x-4|=10$$

$$|x-4|=\frac{10}{3}$$

$$x-4=\frac{10}{3}\text{ or }x-4=-\frac{10}{3}$$

$$\boxed{x=\frac{22}{3},\ x=\frac{2}{3}}$$

(c)

$$x^{2/3}-x^{1/3}-6=0$$

$$\left(x^{1/3}\right)^2-x^{1/3}-6=0$$

$$\left(x^{1/3}-3\right)\left(x^{1/3}+2\right)=0$$

$$x^{1/3}-3=0,\ x^{1/3}+2=0$$

$$x^{1/3}=3,\ x^{1/3}=-2$$

$$x=3^3=\boxed{27},\ x=(-2)^3=\boxed{-8}$$

(d)

$$e^{2x+1}=10$$

$$\ln e^{2x+1}=\ln 10$$

$$2x+1=\ln 10$$

$$2x=-1+\ln 10$$

$$\boxed{x=\frac{-1+\ln 10}{2}}$$

(e)

$$2\ln x=\ln(x+3)+\ln(x-1)$$

$$\ln\left(x^2\right)=\ln[(x+3)(x-1)]$$

$$x^2=(x+3)(x-1)$$

$$x^2=x^2+2x-3$$

$$0=2x-3$$

$$\boxed{x=\frac{3}{2}}$$

6.

(a) For the inequality $x(x-1)^2(x+2) > 0$, first note that the critical numbers (or "cut-points"), that is, the only values of *x* at which the inequality might transition from being satisfied to unsatisfied or vice-versa, are $x = 0,\ x = 1,$ and $x = -2.$

We may choose any value of *x* less than -2 and ask whether the inequality is satisfied for that *x*-value; we find that it is, and therefore that the inequality is satisfied for *all* values of *x* less than -2.

We similarly check a test number between -2 and 0, between 0 and 1, and greater than 1.

We find that the inequality is satisfied when $x < -2$ or $0 < x < 1$ or $x > 1$.

In interval notation: $\boxed{(-\infty,-2)\cup(0,1)\cup(1,\infty)}$

(b) The inequality $|x-4| < 3$ is equivalent to $-3 < x-4 < 3,$ and so $1 < x < 7,$ which in interval notation is $\boxed{(1,7)}$

(c) For the inequality $\dfrac{2x-3}{x+1} \le 1$, rewrite with 0 on the right-hand side and a single rational expression on the left-hand side: $\dfrac{2x-3}{x+1} - 1 \le 0 \rightarrow \dfrac{2x-3}{x+1} - \dfrac{x+1}{x+1} \le 0 \rightarrow \dfrac{x-4}{x+1} \le 0.$

Now note that the critical numbers are $x = 4$ and $x = -1$ and analyze the inequality using test values as in (a). We find that the inequality is satisfied when $-1 < x \le 4,$ which in interval notation is $(-1,4]$.

(Notice here that $x = -1$ itself does *not* satisfy the inequality because it makes the left-hand side undefined, but $x = 4$ *does* satisfy the inequality because it makes the left-hand side 0 and $0 \le 0$.)

7. Let $A(-7,4)$ and $B(5,-12)$.

(a) To find the equation of the line, we first need the slope: $m = \dfrac{-12-4}{5-(-7)} = -\dfrac{4}{3}.$

Now use point-slope form $y - y_1 = m(x - x_1)$ with $m = -\dfrac{4}{3}$ and $(x_1, y_1) = (-7,4)$ to get

$y - 4 = -\dfrac{4}{3}(x-(-7)).$

Solving for *y* and writing the equation in slope-intercept form gives $\boxed{y = -\dfrac{4}{3}x - \dfrac{16}{3}}$

(b) We find the *x*-intercept by setting $y = 0$ and solving for *x* to get $x = -4 \rightarrow \boxed{(-4,0)}$

We find the *y*-intercept by setting $x = 0$ to get $y = -\dfrac{16}{3} \rightarrow \boxed{\left(0,-\dfrac{16}{3}\right)}$

(c) The midpoint of *AB* is $\left(\dfrac{-7+5}{2},\dfrac{4+(-12)}{2}\right) = \boxed{(-1,-4)}$

(d) The length *AB* is $\sqrt{(5-(-7))^2 + (-12-4)^2} = \sqrt{12^2 + (-16)^2} = \sqrt{144+256} = \sqrt{400} = \boxed{20}$

(e) The equation of a circle with center (h,k) and radius r is $(x-h)^2+(y-k)^2=r^2$.
For this circle, the center is the midpoint of the AB, i.e. $(-1,-4)$ and the radius is half of the length of AB, i.e. 10.
Therefore the equation of the circle is given by substituting $h=-1,\ k=-4,\ r=10$ into the equation above to get $\boxed{(x+1)^2+(y+4)^2=100}$

8.

$$\text{If } f(x)=x^3 \text{ then } \frac{f(h+2)-f(2)}{h}=\frac{(h+2)^3-2^3}{h}=\frac{h^3+6h^2+12h+8-8}{h}$$
$$=\frac{h(h^2+6h+12)}{h}$$
$$=\boxed{h^2+6h+12}$$

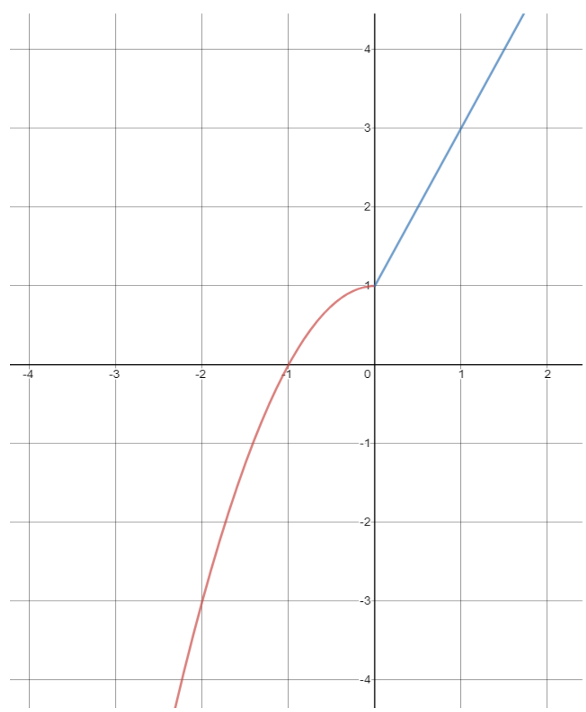
9. (a) $(g\circ f)(x)=g(f(x))=g(x^2+1)=\boxed{\sqrt{x^2+1}}$; The domain is all real numbers.

(b) $(f\circ g)(x)=f(g(x))=f(\sqrt{x})=(\sqrt{x})^2+1=\boxed{x+1}$; The domain is $[0,\infty)$ because in order for x to be in the domain of $f\circ g$, it must in particular be within the domain of g, which is restricted to non-negative numbers (because the square root of a negative number is non-real).

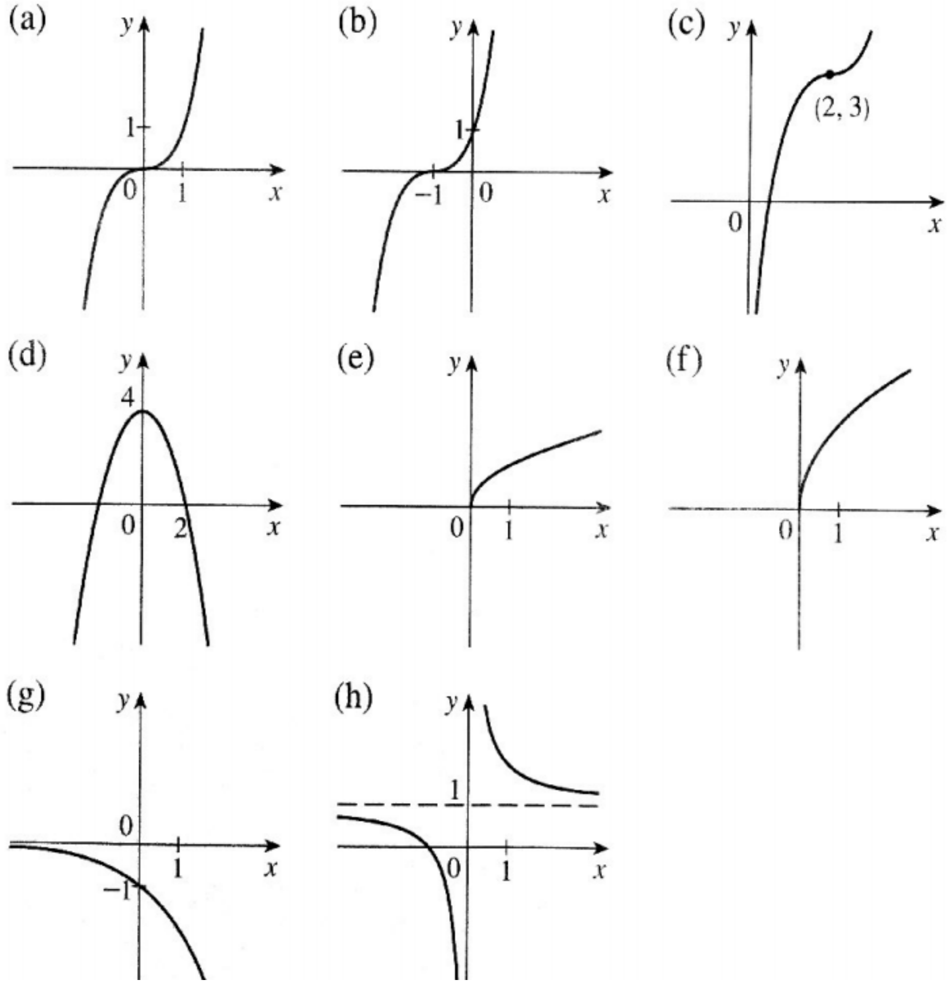
10. (a) The domain is $[-4,8]$ and the range is $[-2,2]$.
(b) The graph is increasing on the interval $(-4,2)$ and $(4,8)$.
(c) $[0,8]$
(d) $f(-2)=2$ and $f(6)=-1$
(e) $x=4$

11. (a) $f(-2)=1-(-2)^2=1-4=\boxed{-3}$;
$f(1)=2(1)+1=\boxed{3}$

(b)

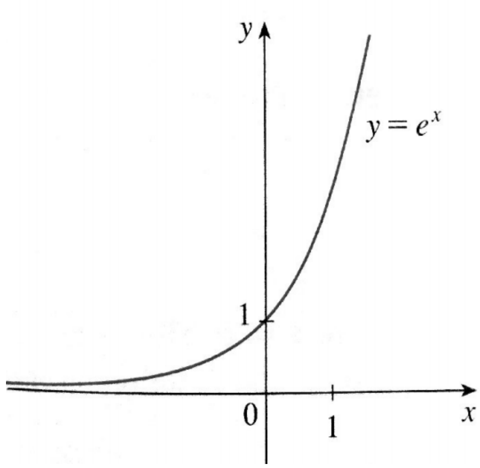


12.



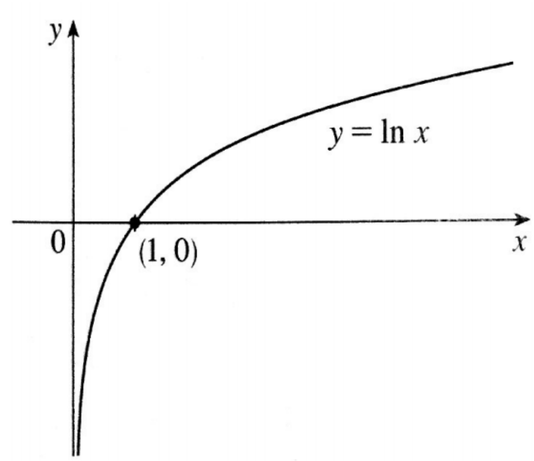
(a)
y
1
0
1
x
(b)
y
1
−1
0
x
(c)
y
(2, 3)
0
x
(d)
y
4
0
2
x
(e)
y
0
1
x
(f)
y
0
1
x
(g)
y
0
1
−1
x
(h)
y
1
0
1
x

(i)



y
$y = e^x$
1
0
1
x

(j)



y
$y = \ln x$
0
(1, 0)
x

Trigonometry Solutions

1. The circumference of a circle of radius r is $2\pi r$, so the circumference of this circle is 10π.

Here, s is $\frac{220}{360}$ of the circumference (noting that $360^\circ - 140^\circ = 220^\circ$).

Therefore, $s = \frac{220}{360}(10\pi) = \boxed{\frac{55\pi}{9}}$

2. Find the values without a calculator.

(a) From memory, $\tan\left(\pi/3\right) = \boxed{\sqrt{3}}$, or, $\sin\left(\pi/3\right) = \frac{\sqrt{3}}{2}$ and $\cos\left(\pi/3\right) = \frac{1}{2}$ from memory and then

$$\tan\left(\pi/3\right) = \sin\left(\pi/3\right) \Big/ \cos\left(\pi/3\right) = \frac{\sqrt{3}/2}{1/2} = \boxed{\sqrt{3}}$$

(b) From memory, $\sin\left(7\pi/6\right) = \boxed{-\frac{1}{2}}$, or, note that the reference angle of $7\pi/6$ is $\pi/6$ and $\sin\left(\pi/6\right) = \frac{1}{2}$ and $7\pi/6$ terminates in Quadrant II where sine is negative, and so $\sin\left(7\pi/6\right) = \boxed{-\frac{1}{2}}$.

(c) $\sec\left(5\pi/3\right) = 1 \Big/ \cos\left(5\pi/3\right) = \frac{1}{1/2} = \boxed{2}$.

3. Using $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ we have $\sin\theta = \frac{a}{24}$ from which $\boxed{a = 24\sin\theta}$.

Using $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ we have $\cos\theta = \frac{b}{24}$ from which $\boxed{a = 24\cos\theta}$.

4. First we'll find the value of $\sin x$. Using $\sin^2 x + \cos^2 x = 1$, we have

$\sin^2 x = 1 - \cos^2 x = 1 - \left(-2/3\right)^2 = 1 - 4/9 = 5/9 \rightarrow \sin x = \pm\sqrt{5/9} = \pm\sqrt{5}/3$.

Because $\pi < x < 3\pi/2$ we have that $\sin x$ is negative, and so $\sin x = -\sqrt{5}/3$.

Now, using $\cos 2x = \cos^2 x - \sin^2 x$ and $\sin 2x = 2\sin x\cos x$ we have:

$\cos 2x = \left(-2/3\right)^2 - \left(-\sqrt{5}/3\right)^2 = \boxed{-1/9}$ and $\sin 2x = 2\sin x\cos x = 2\left(-\sqrt{5}/3\right)\left(-2/3\right) = \boxed{4\sqrt{5}/9}$.

5. By making repeated use of $\cos^2 x = \dfrac{1+\cos 2x}{2}$ we have

$$\cos^4 x = \left(\cos^2 x\right)^2 = \left(\frac{1+\cos 2x}{2}\right)^2 = \frac{1}{4} + \frac{1}{2}\cos 2x + \frac{1}{4}\cos^2 2x$$

$$= \frac{1}{4} + \frac{1}{2}\cos 2x + \frac{1}{4}\cdot\frac{1+\cos 4x}{2} = \frac{1}{4} + \frac{1}{2}\cos 2x + \frac{1}{8} + \frac{1}{8}\cos 4x$$

$$= \boxed{\frac{3}{8} + \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x}$$

6. $\sin 2x = \sin x \to 2\sin x\cos x = \sin x \to 2\sin x\cos x - \sin x = 0 \to \sin x\left(2\cos x - 1\right) = 0$

$\sin x = 0$

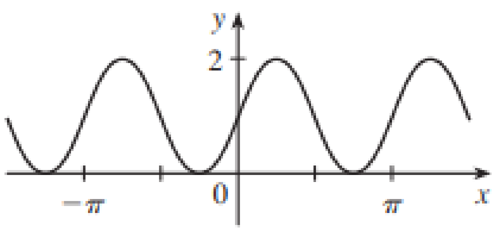
$\boxed{x = 0,\ \pi,\ 2\pi}$

$2\cos x - 1 = 0$

$\cos x = \dfrac{1}{2}$

$\boxed{x = \pi/3,\ 5\pi/3}$

7. The graph of $y = 1 + \sin 2x$ looks like the graph of $y = \sin x$ (which you should know) except that its period has been compressed from 2π to π and the graph is shifted up 1 unit vertically.



8. We recall that $\arccos$ is the inverse cosine function restricted to the range $[0, \pi]$ and that $\arcsin$ is the inverse sine function restricted to the range $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$.

(a) $\arccos\left(-\dfrac{\sqrt{2}}{2}\right) = \boxed{\dfrac{3\pi}{4}}$ because $\cos\dfrac{3\pi}{4} = -\dfrac{\sqrt{2}}{2}$ and $\dfrac{3\pi}{4} \in [0, \pi]$.

(b) $\arcsin\left(\sin\dfrac{5\pi}{4}\right) = \arcsin\left(-\dfrac{\sqrt{2}}{2}\right) = \boxed{-\dfrac{\pi}{4}}$ because $\sin\left(-\dfrac{\pi}{4}\right) = -\dfrac{\sqrt{2}}{2}$ and $-\dfrac{\pi}{4} \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$.

Note that the answer is not $\dfrac{5\pi}{4}$ because, by definition, the range of arcsin is restricted to $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$ and $\dfrac{5\pi}{4}$ is not in this interval.

(c) Let $\theta = \arccos\frac{3}{4}$. Then $\cos\theta = \frac{3}{4}$. Now $\tan\left(\arccos\frac{3}{4}\right) = \tan\theta$, and from the identity $1+\tan^2\theta = \sec^2\theta$ (which comes from $\sin^2\theta + \cos^2\theta = 1$), we have

$$\tan\theta = \sqrt{\sec^2\theta - 1} = \sqrt{\frac{1}{\cos^2\theta} - 1} = \sqrt{\frac{1}{\left(\frac{3}{4}\right)^2} - 1} = \sqrt{\frac{7}{9}} = \boxed{\frac{\sqrt{7}}{3}}$$

9. By Law of Cosines,

$$\begin{aligned} 7^2 &= 3^2 + 5^2 - 2\cdot 3\cdot 5\cos C \\ 49 &= 34 - 30\cos C \\ 15 &= -30\cos C \\ -\frac{1}{2} &= \cos C \\ &\boxed{C = 120^\circ} \end{aligned}$$

After reading through the solutions, which of the following most closely describes you?

A. I am quite comfortable with the skills from both sections (College Algebra and Trigonometry).

B. I am comfortable with a lot of the skills from both sections (College Algebra and Trigonometry) but some of the problems from one or both categories would be difficult for me to complete. I could use review on a lot of these topics.

C. I am quite comfortable with the skills from the first section (College Algebra) but I am quite uncomfortable with a lot of the skills from the second section (Trigonometry).

D. I am uncomfortable with a lot of the skills from both sections (College Algebra and Trigonometry).

If you chose **A** then you should **Enroll in Math 150**.

If you chose **B** then you should **Enroll in Math 150 along with Math 150C, a 2-unit Support Course**.

If you chose **C** then you should **Enroll in Math 138**.

If you chose **D** then you should **Enroll in Math 137 or below.** See the Placement Guide for Math 137: http://sbcc.edu/assessmentcenter//files/137placementguide.pdf